DC power system stability

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Abstract - The advantages of utilising direct current electricity for the transmission of power has many acknowledged advantages and it is now receiving serious consideration for adoption in the marine and naval sectors, indeed manufacturers of marine electrical propulsion systems are confident of offering DC power systems to the commercial market in the near future and design studies for future naval platforms in many of the world's navies are favouring DC. But in addition to the manifold advantages such as power transmission density, efficiency and flexibility there is one inherent weakness - stability when supplying constant power loads - which whilst not an incurable problem first needs to be understood in order that a robust design can be achieved in all operating conditions. The problem is highly non-linear and has resulted in several papers applying complex and highly arcane methodologies. However in the authors' view the problem is tractable to the ubiquitous control analysis of linearisation about a set point - even though the problem itself being a physical manifestation of hardware characteristics is not strictly within the control engineer's domain. This paper explains the source of the instability illustrates the analysis methodology, assesses a method of compensation and compares the linearised approach to a non-linear approach.

I. INTRODUCTION

DC has long been recognised as a medium that offers efficient and effective power transmission and as such offers significant advantages over its counterpart of AC electricity. Hodge and Mattick, [1] and [2], explain much of the reasoning for favouring DC as a power transmission medium and the general view of the maritime electrical power industry is that DC will be adopted for many of the foreseeable applications in the commercial and naval marine environments in the near future.

However there is one area where DC power systems need to be treated with some care and that is when they supply constant power loads. In this situation current rises as voltage falls (and vice versa) and this is, intuitively, a situation that may lead to instability.

II. THE CONSTANT POWER STABILITY PROBLEM

As mentioned above the situation where a power system feeds a constant power load may suffer from instabilities. This is because a falling voltage leads to an increase in current and that itself reduces voltage further due to the supply system impedance. However the situation can be analysed readily by control engineering techniques.

In doing so it needs to be remembered that the system itself is not a control system nor is there any need for it to be governed by a control system that itself seeks to maintain power constant. However the equations that arise from the analysis are identical in mathematical terms to those that would arise from a standard control system. This enables - once the similarity is established - the full gamut of the control engineer's analytical methods to be applied.

As will be shown later the system, in its abstract form as a constant power load, is deceptively simple; but it is also important to realise that there are many examples of practical constant power loads that arise in many power systems. The simplest are constant speed fluid pumps that are driven by a supply of fixed frequency. This may be considered as a converter fed pump where the frequency of the supply, (perhaps inverted from a DC link) is controlled to a constant without reference to the power drawn by the pump. This decouples the power the pump draws from the supply voltage. Thus, even though the voltage may fall, the speed of the pump remains constant (as frequency does not change) and as a result the pump alters its phase relationship to the supply and thereby draws more current and maintains constant power.

The problem as analysed below is non-linear and Sudhoff et al [3] demonstrate a non-linear analysis of a constant power DC system. However the analysis does not provide a better design methodology than the simpler linearisation process illustrated here.

III. THE ANALYSIS

A. *Introduction*

The next stage is to analyse the theoretical basis for a DC systems constant power instability. The following applies control engineering analysis to the stability of a simplified, linearised and compensated constant power DC circuit.

B. *Characteristic Equation*

The circuit diagram of interest - including the necessary capacitance for stability - is shown at Figure 1. This also includes the characterisation of the impedance as a negative resistance, which is correct for perturbational quantities. The circuit at Figure 1 can be used to derive a equation that is directly comparable to a control systems characteristic Once the characteristic equation is known the stability of the circuit can be analysed with respect to the variation of any of its principal parameters - for example the stabilising capacitance or the load resistance. The characteristic equation for Figure 1 can be rapidly developed by writing the circuit components in their Laplace form and applying any of the common methods of circuit analysis (Millman's Theorem for example) and this results in the following "closed loop" transfer function:

$$\frac{v}{E} = \frac{1}{CLs^2 + \left(Cr - L/R\right)s + \left(1 - r/R\right)}$$

Which then implies the characteristic equation as:

$$CE = CLs^2 + \left(Cr - L/R\right)s + \left(1 - r/R\right) = 0$$

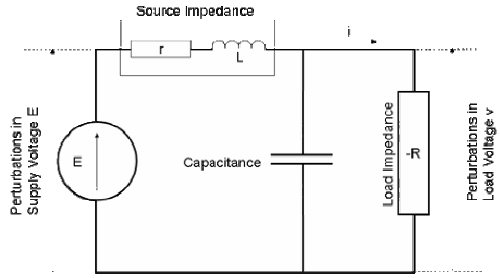


Figure 1: DC Constant Power System

With the governing equation known there is a variety of analysis techniques that can be applied. But first it is worth noting that equation (1) governs the actual perturbational response of the circuit to some disturbance whilst operating at any desired set point. As such it must be viewed as the closed loop transfer function and the root locus technique is therefore directly applicable.

As an example consider the following circuit parameters in Table 1:

Parameter	**Value**	**Units**
r	300	mΩ
L	10	mH
R	24.3	Ω

Table 1: Circuit Parameters

This system would deliver 3.7 kW at 300V, under constant power conditions.

C. *Control Engineering Techniques*

Routh-Hurwitz may be used to determine the minimum value of the capacitance for stability or inequalities may be used derived directly from the circuit parameters and the need to retain negative exponential exponents in the time domain solution. Using these inequalities:

R>r is satisfied.

The critical value of C is therefore given by,

$$C = \frac{L}{rR} = 1.37\text{mF}$$

The stability of the circuit can be assessed through application of the root locus technique and this is shown in Figure 2 & Figure 3. The criteria for stability in the root locus technique is that the roots to the characteristic equation must have negative real parts (in order to ensure that the transient decays). The range of C varies from zero (uncompensated) to infinity (in practice just very large). The time domain response of the systems was investigated at seven specific points as given in Table 2.

Root Position No	Capacitor Value	Units
1	0.44	F
2	36.5	mF
3	8.1	mF
4	4.1	mF
5	2.0	mF
6	1.4	mF
7	1.0	mF

Table 2: Values of Capacitance for Figure 3

As Figure 2 shows the root locus is mostly in the unstable zone - that is to the right of the y axis. However there is a stable area to the left and this is shown in expanded form in Figure 3 The seven points indicated on Figure 3 show the points from Table 2. The system responded as predicted with instability imminent at point 6 and fully developed at point 7.

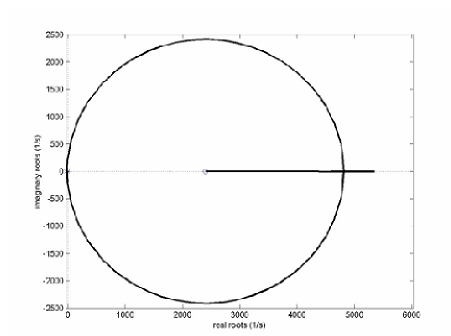


Figure 2: Full Root Locus

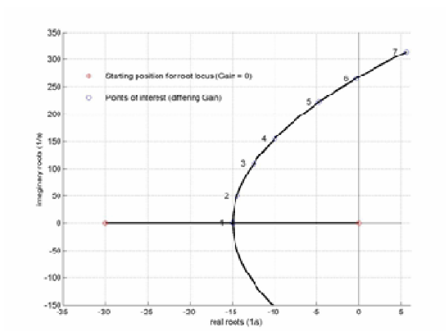
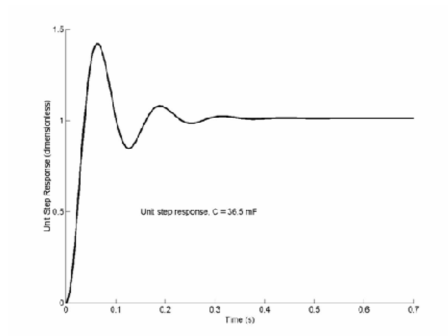


Figure 3: Detail of Root Locus

The time domain response can be derived at any operating point on the constant power characteristic although the limitations of the process of linearisation around a set point need to be borne in mind when this is done. Figure 4 illustrates the time domain response for the circuit parameters as defined at Table 1 and for a stabilising capacitance value of 36.5 mF.

Other control engineering techniques are possible in addition to the time domain method of root-locus the stability can be investigated in the frequency domain by application of Bode and Nyquist but these techniques apply to the open loop transfer function rather than the closed loop characteristic function equation

Figure 4: Time Domain Response for 36.5mF



IV. COMPENSATION

Reference [4] is a paper, by Flower and Hodge due to be published by the IMarEST and it deals with compensation of the circuit in order to achieve additional margins of stability. The problem is not a simple one. The dominant determinants of stability are the load resistance, the source impedance (Source resistance and source inductance) and the stabilising capacitance. There are limits on increasing the stability margin by altering each of these. The load resistance cannot be increased else the supply would be providing insufficient power, the source impedance cannot be reduced as it is already minimised for supply efficiency. Thus it is normally capacitance that is increased to ensure stability - albeit this is far from desirable as it thereby increases cost, volume and mass.

However there is a little seen or mentioned alternative which is fully explored in Reference [4] and consists of placing a small series resistance, of the order of Ohms, in the parallel capacitive branch in series with the stabilising capacitor. Placing the resistance in the parallel capacitive branch removes the stabilising resistance from the power circuit and thus no additional power is dissipated under steady sate conditions and in addition the power dissipated during transients when the capacitor branch enters play is minimal because the new stabilising resistance is itself relatively small. The revised compensated circuit is at Figure 5.

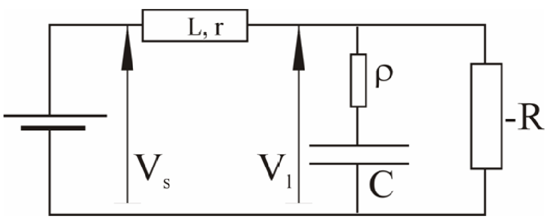


Figure 5: Circuit for Resistive-Capacitive Compensation

The overall transfer function can be derived in a similar way to that done previously for the uncompensated and capacitive-compensated cases (Appendix 2). The result is:

$$\frac{V_o(s)}{V_s(s)} = \frac{R(Cs+1)}{CL(R-\rho)s^2 + \left[RC(\rho+r) - C\rho r - L\right]s + (R-r)},$$

And from which the Characteristic Equation can again be determined:

$$CE = CL(R-\rho)s^2 + \left[RC(\rho+r) - C\rho r - L\right]s + (R-r)$$

The corresponding Routh-Hurwitz array is:

s^2	$CL(R-\rho)s^2$	$(R-r)$
s^1	$\left[RC(\rho+r)-C\rho r-L\right]$	0
s^0	$(R-r)$	

From which and knowing that all the component values are positive, the requirements for stability are:

$$R \rangle \rho$$

$$R \rangle r$$

$$RC(\rho+r) \rangle (C\rho r + L).$$

Interestingly enough, it can be seen that if r were negligible, then using appropriate values of C and ρ the circuit can be made stable, conversely without the compensating resistor ρ the condition of r being equal to zero will always be unstable if the compensating capacitor is retained. Suppose, in fact, that in the above, r is zero, then the conditions for stability become,

$$R \rangle \rho$$

and

$$RC\rho \rangle L.$$

Effectively, under this condition, ρ replaces r in the original stability conditions with r present (but without ρ). But physically no power is dissipated under dc conditions in the compensating resistance ρ and the load is subjected to a voltage equal to the supply EMF.

In a given design situation R, L and r will be pre-specified, but the designer now has two variables, C and ρ, to choose in order to satisfy the inequalities governing stability. And, if possible, improve the transient behaviour also.

Note, in equation (3), that the addition of ρ has led to the appearance of a zero in the transfer function, $\frac{V_o}{V_s}$, thus the dynamic behaviour is more complex, although the order of the system remains two. This latter fact is expected since no further energy storage-type components have been added to the circuit.

In the following it is shown that this circuit, with appropriate values of ρ and C, can lead to significant improvements in the stability and in transient behaviour. The additional power, consumed by ρ, being trivial compared with the power delivered to the load.

D. *Example of Resistive-Capacitive Compensation*

If r = 0.3Ω, L = 10mH, C = 4.1 mF, R = 24.3Ω and ρ is varied, then the function defining the root-locus is,

$$\frac{\rho s\left(-4.1\text{x}10^{-5}s + 9.85\text{x}10^{-2}\right)}{\left(9.97s^2 + 1\text{x}10^{-2}s + 25\right)}$$

The function has a zero at the origin and in the right-half plane; it has two complex-conjugate poles in the left-half plane, quite close to the imaginary axis. Instability at high values of ρ is to be expected - as ρ increases in magnitude it limits the ability of the capacitor to influence the dynamics. In control terms this would be termed a ‘conditionally-stable’ system.

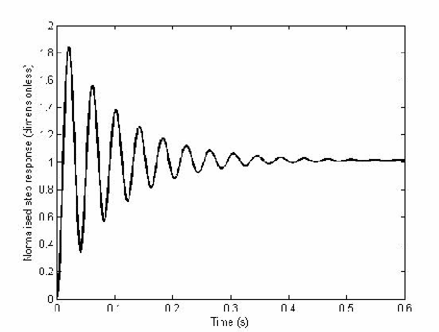


Figure 6: Time Domain Simulation ρ = 0

Figure 6 illustrates the response to a unit-step change in the supply voltage, with the compensating resistor, ρ, set to zero.

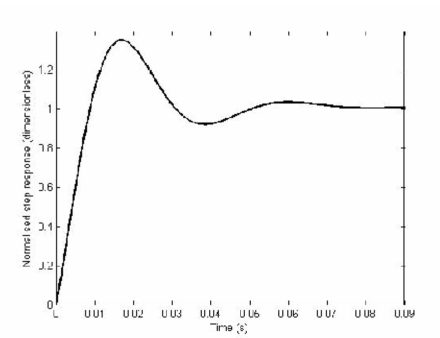


Figure 7: Time Domain Simulation ρ Equal to 1Ω

Conversely Figure 7 shows the that the response changes dramatically when ρ is equal to 1Ω and is illustrated in, the transient has died away in a little over 0.07s, almost an order of magnitude less than the response obtained for ρ equal to zero. The maximum voltage-overshoot is some 34%.

V. NON-LINEAR CONSIDERATIONS

In this section some preliminary findings from a non-linear study are presented together with an assessment of how useful the corresponding linear results may be when large step-changes are made in the supply voltage. Those results are best appreciated in graphical form and Simulink has been used throughout to obtain them.

The mathematic description of the load characteristic is vi=P (a constant) and represents a rectangular hyperbola and is a non-linear relationship. The results presented earlier are all based on linear techniques and may be expected to provide accurate information for minor perturbations. There is a need therefore to investigate what use these techniques may, or may not be, when dealing with a patently non-linear system.

Figure 8 and Figure 9 show the voltage across the load using a non-linear model and a linear model respectively. These responses had been obtained from the condition where there has been a step-change in the supply voltage, E, from 280V to 350V.

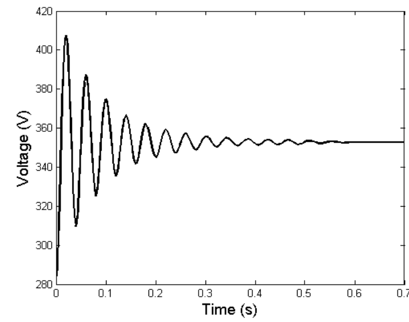


Figure 8: Step Up Voltage Response – Non-Linear Model

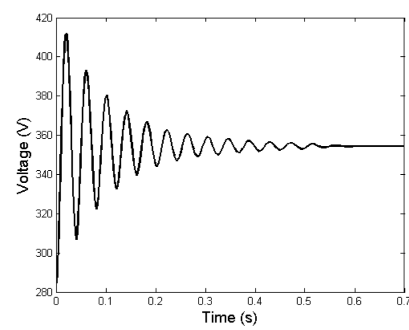


Figure 9: Step Up Voltage Response – Linear Model

In the case of the linear responses the question arises as to which value of R (the "dc" resistance) should be used. The value of R at 280V is 19 Ohms, but at 350V it is 32 Ohms. The voltage responses using these two values are shown in Figure 10 and Figure 11 respectively. The responses are agreeably similar, the lower value is slightly more oscillatory and is damped out less rapidly.

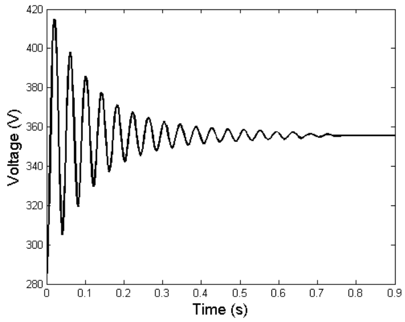


Figure 10: Step Up Voltage Response – Linearised Model – R = 19 Ohms (280V)

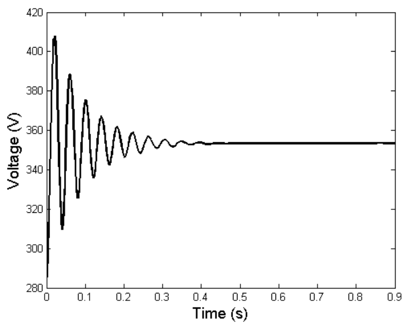


Figure 11: Step Up Voltage Response – Linear model – R = 32 Ohms (350V)

The earlier part of the response is closer to the response obtained from the higher value of R. These results, over the range studied indicate that very good estimates of behaviour can be obtained using a linear treatment.

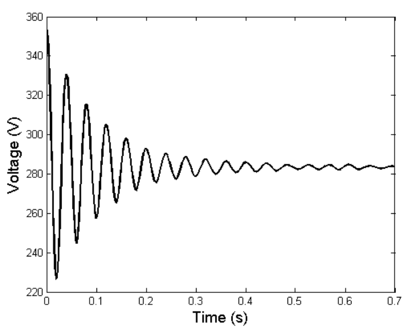


Figure 12: Step Down Voltage Response – Non-Linearised Model

Figure 12 shows the non-linear response for the supply voltage step-changing from 350V to 280V. Even allowing for the fact that the voltage change is in the other direction, it can be seen that the response is somewhat different to the characteristic for the equivalent step up at. The voltage oscillation dies more rapidly for the step up disturbance. That these bi-directional responses differ indicates that the non-linear behaviour is perceptible. This would not be the case for a bi-directional response in a linear treatment using the same value for R.

The authors are still examining aspects of the non-linear behaviour and will report the findings in due course.

VI. CONCLUSION

The origin of DC power system constant power instability has been explained and its highly non-linear nature emphasised. The advantages of applying a control engineering analysis to this essentially electrical power engineering problem have been demonstrated as has the process of linearisation around a set point. The advantages of adopting this traditional analysis technique are considerable and allow accurate assessment of the boundaries of stability and provide useful data with respect to the dynamics of the system's time domain performance.

It has been demonstrated that by using a resistor of small value in series with the compensating capacitor the stability of the demonstration example can be improved with great effect. The corresponding transient-response can be dramatically improved with respect to its form and to its time to settle. Although this resistor is power consuming its effect on energy-efficiency is trivially small. This implies that this resistor does not need special cooling arrangements to be made.

An interesting observation is that, although this arrangement introduces a zero into the transfer function (between load voltage and the input voltage), this zero improves the transient performance which is most agreeable.

A comparison has been made between the results derived form a linearised model and a non-linear model.

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